Name: ______________________

Row: ______________________

Math 113H, Section 12
Exam 1

Instructor: David G. Wright

16-18 September 2010

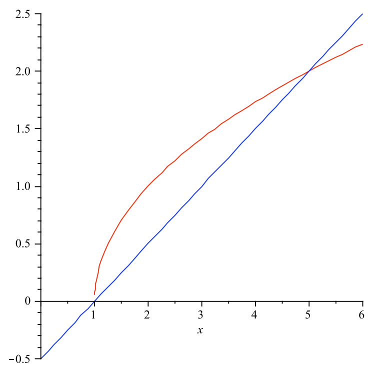
Instructions:

1. Work on scratch paper will not be graded.
2. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
3. Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
4. Calculators are not allowed.

For Instructor use only.

#	Possible	Earned		#	Possible	Earned
1.a	6			4	10	
1.b	6			5.a	8	
1.c	6			5.b	8	
1.d	6			5.c	8	
1.e	6			5.d	8	
2	10			5.3	8	
3	10			Total	100	

1. (30%) Consider the region between the curves $y = \sqrt{x-1}$ and $y = \dfrac{x-1}{2}$.



(a) Set up an integral for the area of the region bounded by the curves. DO NOT SIMPLIFY. DO NOT EVALUATE.

$$\int_1^5 \left(\sqrt{x-1} - \frac{x-1}{2}\right) dx$$

(b) Use the Washer Method to set up an integral for the volume when the region is rotated about the $x-$axis. DO NOT SIMPLIFY. DO NOT EVALUATE.

$$\pi \int_1^5 \left((x-1) - \left(\frac{x-1}{2}\right)^2\right) dx$$

(c) Use the Shell Method to set up an integral for the volume when the region is rotated about the $y - axis$. DO NOT SIMPLIFY. DO NOT EVALUATE.

$$2\pi \int_1^5 x\left(\sqrt{x-1} - \frac{x-1}{2}\right) dx$$

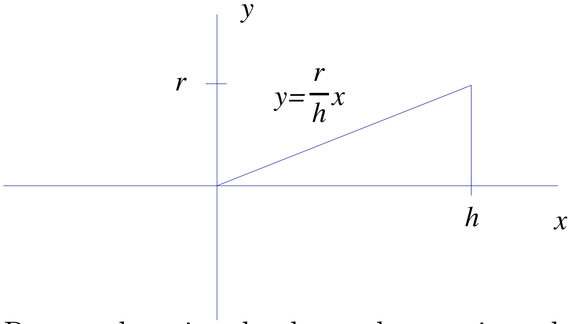
(d) Set up an integral for the volume when the region is rotated about the line $y = 2$. DO NOT SIMPLIFY. DO NOT EVALUATE.

$$\pi \int_1^5 \left[\left(2 - \frac{x-1}{2}\right)^2 - \left(2 - \sqrt{x-1}\right)^2\right] dx$$

(e) Set up an integral for the volume when the region is rotated about the line $x = -1$. DO NOT SIMPLIFY. DO NOT EVALUATE.

$$2\pi \int_1^5 (x+1)\left(\sqrt{x-1} - \frac{x-1}{2}\right) dx$$

2. (10%) Use the disk method or the shell method to show that the volume V of a cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.



Rotate the triangle about the x-axis and use the disk method.

$$\pi \int_0^h \left(\frac{r}{h}x\right)^2 dx = \pi \frac{r^2}{h^2}\frac{x^3}{3}\Big|_0^h = \frac{1}{3}\pi r^2 h$$

3. (10%) A bucket that weighs 4 lb and a rope that weighs 0.2 lb per foot are used to draw water from a well that is 50 ft deep. The bucket is filled with 40 lb of water and is pulled up at a constant speed, but water leaks out of a hole in the bucket at a constant rate so that only half the water reaches the top. Find the work done in pulling the bucket to the top of the well.

Work for rope $= \displaystyle\int_0^{50} \frac{1}{5} x dx = 250$ ft-lbs

Work for the 20 lbs of water that doesn't spill $= 20 \cdot 50 = 1000$ ft-lbs

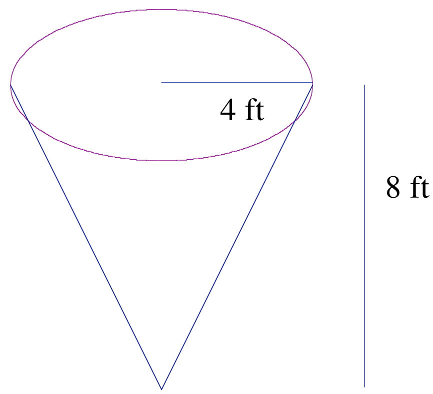
Work for bucket $= 4 \cdot 50 = 200$ ft-lbs

Work for water that spills. Weight of water at distance x from the top $= \dfrac{2}{5}x$.

$\displaystyle\int_0^{50} \frac{2}{5} x dx = 500$ ft-lbs

Total work $= 1950$ ft-lbs

4. (10%) A conical tank of radius 4 ft and height 8 ft is full of water of density 62.5 lbs per ft^3. Set up an integral that represents the work in foot pounds needed to pump the water to a height 2 ft above the top of the tank.



$$62.5\pi \| int_0^8 (10 - y) \left(\frac{y}{2}\right)^2 \, dy$$

5. (40%) Evaluate the following integrals. Show your work.

   (a) $\int (\ln x)^2 \, dx$

   Use Integration by Parts twice. Let $u = (\ln x)^2$ and $dv = dx$ to get $x(\ln x)^2 - 2\int \ln x dx$ Apply integration by parts to the simplified integral with $u = \ln x$ $dv = dx$ to get the final answer of
   $x(\ln x)^2 - 2x\ln x + 2x + C$

   (b) $\int_0^{\pi/4} \cos^2(2x) \, dx$

   $$= \int_0^{\pi/4} \frac{1+\cos(4x)}{2} dx = \frac{x}{2}|_0^{\pi/4} + \frac{\sin(4x)}{8}|_0^{\pi/4} = \frac{\pi}{8}$$

(c) $\int t \cos t \; dt$

Use integration by Parts with $u = t$ and $dv = \cos t$

$$t \sin t + \cos t + C$$

(d) $\int_0^{\pi/4} \sec^3 \theta \; d\theta$

Let I be the given definite integral. Use Integration by Parts with $u = \sec\theta$ and $dv = \sec^2\theta d\theta$. The new integral has a $\tan^2\theta$ that can be replaced by $\sec^2\theta - 1$. The resulting equation can be solved for

$$2I = \sec\theta\tan\theta\Big|_0^{\pi/4} + \int_0^{\pi/4} \sec\theta \; d\theta$$

The final solution is $\dfrac{\sqrt{2} + \ln(\sqrt{2}+1)}{2}$

(e) $\int \tan^2 x \sec^4 x \; dx$

$$= \int \tan^2 x \sec^2 x \sec^2 x \; dx = \int \tan^2 x (\tan^2 x + 1) \sec^2 x \; dx =$$

$$\int (\tan^4 x + \tan^2 x) \sec^2 x \; dx = \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + C$$